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APPLICATION OF THE EIGENFREQUENCY OF A CONVECTIVE CELL TO DETERMINATION OF THE FREQUENCY CHARACTERISTICS OF OSCILLATORY THERMAL GRAVITATIONAL AND THERMOCAPILLARY CONVECTION IN CLOSED VOLUMES

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The notion of the eigenfrequency of a convective cell has been introduced and three methods for determination of this parameter in numerical experiments have been proposed. The effect of resonance of the stream function in the field of a rotating gravity vector (this effect has been found earlier) is used for thermal gravitational convection in a square cavity. Thermocapillary convection is investigated in a rectangular cavity and in a cylinder. In these cases use is made of the resonance effect and the variation of the parameters with eigenfrequency under the action of a heat flux harmonically oscillating and acting on the free surface of the liquid. The correspondence between the obtained eigenfrequencies of the convective cell and the frequencies of oscillatory convection, obtained in numerical calculations and experiment, has been established.

Introduction. The problem of turbulent convection regimes has long been the focus of attention of theoretical hydromechanics but, despite this fact, it remains to be finally solved. A specific case of turbulent flows is the oscillatory regime of convection in closed liquid volumes. Unlike the classical turbulence in the case of external flow past solid surfaces, such convection regimes are characterized by the ordered variations of the parameters of a liquid whose spectrum contains a finite number of frequencies. The oscillatory convection regime has generated great interest in recent times in connection with important technical applications, primarily with the processes of growth of bulk single crystals. Determination of the frequency spectrum in the oscillatory convection regime and their relations to the geometric characteristics of liquid volumes, the dynamic parameters of flow, and the boundary conditions is an important problem for construction of the theory of stability of convection in closed volumes.

The relation of temperature variations in a melt to the microsegregation of active impurities in the grown crystals with the so-called impurity striation of crystals has been established in [1, 2]. Impurity striation is a very significant defect of a crystal which is expected to be used in microelectronics, for example, as a substrate in creation of large-scale integrated circuits. Therefore, we have seen many publications appearing on the oscillatory regime of thermal gravitational convection and on the methods of suppression of temperature oscillations in a melt. Works on oscillating thermal gravitational convection have been reviewed in [3].

It has been found in experiments on crystal growth aboard spacecraft that for the case where we have a free nonisothermal melt surface (constantly existing or periodically formed), of importance is thermocapillary convection that finds itself in the region of oscillatory regime in crystal growth by the floating-zone method (liquid column with a free cylindrical surface) under microgravity conditions. The number of experimental works on investigation of oscillating thermocapillary convection is small. The experiments under microgravity conditions were carried out only for liquids with large Prandtl numbers ($\mathrm{Pr} > 1$), whereas data for liquids with $\mathrm{Pr} < 1$ (melts of semiconductor materials) are critically required. In the analysis of the experimental results, the action of vibration was disregarded, although this action is significant for free liquids, as our investigations [4, 5] have shown. Works on oscillating thermocapillary convection have been reviewed in [6]. It is noted that even a low azimuthal velocity in a cylindrical volume of a liquid

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(three-dimensional flow) causes the oscillations to appear much earlier than in the case of the axial symmetry of flow (two-dimensional approximation).

Different modes of temperature variations (oscillations) — harmonic oscillations [1] and nonharmonic ones [2] — were obtained as early as in the pioneering works on oscillating thermal gravitational convection. The reason for such a difference was not quite clear until very recent times, since the mechanism of occurrence of the oscillations was not clear.

1. Formulation of the Problem and Procedure of Solution. Numerical calculations of oscillating convection in a two-dimensional formulation [4] clearly show that the oscillations are related to the periodic change in the shape and location of convective cells in a liquid. This has allowed the conclusion [8] that the spectra of oscillatory regimes are the reflection of ordered space-time vortex structures in the liquid. In this work, the notion of the eigenfrequency of a convective cell was used as one characteristic of these structures. Thereafter it was assumed that the frequency of oscillations of the parameters in the liquid is identical to the eigenfrequency of the convective cell. Let us consider the formulation of the problem on determination of the eigenfrequencies of convective cells and procedures of its solution separately for both types of convection in two-dimensional flows.

Thermal Gravitational Convection. The driving force for thermal gravitational convection is the buoyancy force which contains free fall acceleration as one parameter in the expression for this force. Therefore, in this case it seems natural to apply one type of harmonic oscillations of free fall acceleration or another to the determination of the eigenfrequencies of convective cells. Measurements of microaccelerations aboard the Salyut orbital station gave impetus to the beginning of investigations in the field of space-time evolutions of the gravity vector [9]. In the analysis of the measurements, it was established that the vector of vibrational microaccelerations rotates in a plane. For thermal gravitational convection we found the resonance effect where the frequency of the indicated evolutions of the acceleration vector attains a certain value (F_{res}) [10, 11]. The resonance effect is attained not only in rotation of the acceleration vector in a plane but also in its harmonic linear oscillation with the same frequency perpendicularly to the prescribed temperature gradient. We obtained resonant frequencies for a liquid with Prandtl number Pr = 1 in [10, 11] and for Pr = 0.018 in [8]. It should be noted that the vibrational frequencies aboard the spacecraft lie in the region of much higher frequencies than the eigenfrequencies for convective cells. Furthermore, the value of vibrational accelerations aboard is too low for us to observe the effect directly aboard the spacecraft. Therefore, a special experiment on checking it aboard the Mir orbital space station was proposed ten years ago. It was assumed to simulate the the rotation of the vector of gravitational acceleration in zero gravity using centrifugal acceleration in the rotation of a square cavity with frequency F_{res} and synchronous rotation of the cavity with a liquid about its center. The condition implying that the radius of rotation of the cavity is much larger than half the side of the square enables us to disregard the acceleration produced by the rotation of the cavity. The phenomenon of resonance found in the numerical calculations is an example of virtual numerical experiments that can be used as an efficient means for studying fundamental phenomena in fluid mechanics, when a physical experiment is in principle impossible or difficult to realize.

Thermal gravitational convection is investigated for a square region. One boundary is heat-insulated. A linear temperature distribution between the values T_0 and T_{max} is prescribed on the opposite side. The temperatures T_0 and T_{max} on the other two sides are held constant. The vector of free fall acceleration is arranged on the plane in various manners depending on the problem solved. In the calculations of the oscillating convection regime, the acceleration is constant in value, acts in parallel to the boundaries with prescribed values of the temperature, and is directed toward the heat-insulated boundary. If we solve the problem of finding the resonance, the acceleration vector rotates in the plane in question about its center with a prescribed angular frequency $\omega = 2\pi f$. All the boundaries of the region represent solid walls on which the adhesion condition is set. The liquid (fluid) is viscous and incompressible.

The dimensionless equations that control the transfer of momentum and heat in the liquid are written in vector form

$$\mathrm{div}\, \mathbf{v} = 0 \,, \tag{1}$$

$$\partial \mathbf{v} / \partial \tau + (\mathbf{v} \Delta)\, \mathbf{v} = - \nabla p + \nabla^2 \mathbf{v} + \theta \mathbf{e}_g \,, \tag{2}$$

$$\partial \theta / \partial \tau + (\mathbf{v} \Delta)\, \theta = \mathrm{Pr}^{-1}\, \nabla^2 \theta \,, \tag{3}$$

where $\mathbf{e}_g(-\mathrm{Gr}_{T,x};\ 0)$ is used for calculations of oscillating convection and $\mathbf{e}_g(\mathrm{Gr}_{T,x} \sin(\Omega, \pi) + \mathrm{Gr}_{T,y}(\Omega, \tau))$ is used for investigation of the phenomenon of resonance.

Thermocapillary Convection. Thermocapillary convection occurs in the presence of the pressure gradient on the free surface of a liquid and is determined by the value of this gradient and by the temperature dependence of the surface tension. This phenomenon is of importance in a number of technological processes but is particularly pronounced under microgravity conditions. The floating-zone method is the most promising for microgravity conditions; therefore, we modeled thermal conditions precisely for this method of growing crystals. In this method, a distributed heat flux with a maximum at the center of the length of a liquid zone acts on its free surface. To determine the eigenfrequency of a convective cell under the thermocapillary effect it was logical to use the harmonic action on the temperature field on the free liquid surface. We used two methods of such an action. The first method uses a comparatively small harmonically oscillating heat flux; this flux is superimposed on a constant heat flux arriving at the free surface of the liquid. It is assumed that when the oscillation frequency of the variable heat flux coincides with the eigenfrequency of the convection cell, we must have a resonance and the stream function will attain its maximum value. The second method differs from the first one in that the additional heat flux is fairly large but short-acting, i.e., a shock thermal action occurs. In this case one might expect that after the transient period, damped oscillations of the parameters will be executed at the eigenfrequencies of the convective cells.

Thermocapillary convection is considered for rectangular or cylindrical regions with elongation L/d or L/R respectively. One long side of the rectangle or the lateral cylindrical surface represents the free liquid surface on which the external heat flux is prescribed in the form of the exponent

$$(\partial\theta/\partial r;\ x)\big|_{r,x=1} = A \exp\left[-B\,(z;\ y - L/2)^2\right] . \tag{4}$$

Such a time-constant heat flux acts when the oscillating regime of convection is studied. If the eigenfrequency of a convective cell is determined, the total heat flux takes the following form:

$$(\partial\theta/\partial r;\ x)\big|_{r,x=1} = A \exp\left[-B\,(z;\ y - L/2)^2\right] + A_1 \exp\left[-B\,(z;\ y - L/2)^2\right] \sin(\Omega\tau) . \tag{5}$$

The opposite side represents the axis of symmetry for the cylinder or the heat-insulated solid wall for the rectangle. A constant initial temperature T_0 is prescribed on the other two sides. Equations describing flow and heat transfer in the case of action of thermocapillary convection are generally the same as those in studying gravitational convection. The difference is that we consider the condition of total zero gravity ($\mathrm{Gr}_T = 0$) and, instead of the adhesion condition at all the boundaries, we use in this case the kinematic condition for the tangential velocity component on the free liquid surface (r, $x = 1$), which describes the thermocapillary effect:

$$\partial v/\partial\tau = -\,\mathrm{Ma}\,\mathrm{Pr}^{-1}\,\partial\theta/\partial z\,(y) . \tag{6}$$

In obtaining the dimensionless equations (1)–(6) and the dimensionless parameters, we used the following scales: R or d for the linear dimension, νR^{-1} (or νd^{-1}) for the velocity, $R^2\nu^{-1}$ (or $d^2\nu^{-1}$) for the time, $R^{-2}\nu$ (or $d^{-2}\nu$) for the frequency, and $\rho\nu^2R^{-2}$ (or $\rho\nu^2d^{-2}$) for the pressure. The temperature difference is prescribed in the problem on thermal gravitational convection and is not prescribed for thermocapillary convection, since, according to the condition of the problem, the heat flux at the free boundary acts. However, the Marangoni number was corrected by multiplication by the maximum temperature (θ_{max}) actually obtained in the system.

Difference Scheme. Since calculations were assumed to be carried out in the region of fairly high velocities of flow of the viscous liquid, the requirements imposed on the difference scheme must be high. To solve the problem formulated we used a finite-difference scheme of the third order of accuracy in spatial coordinates with clustering of the grid nodes toward the boundaries of the computational domain, where we have high velocities or large gradients of velocity of liquid flow. The scheme is implicit and conservative and possesses the property of conditionally monotone approximation of the convective term in the equation of motion (2). The efficient computational algorithm developed on the basis of the implicit conjugate-gradient method possesses rapid convergence. In calculating pressure, we determined the increment of pressure. The finite-difference scheme is checked in the well-known test on liquid flow in a three-dimensional cavern with a moving cover. A comparison of the calculations according to this procedure with the

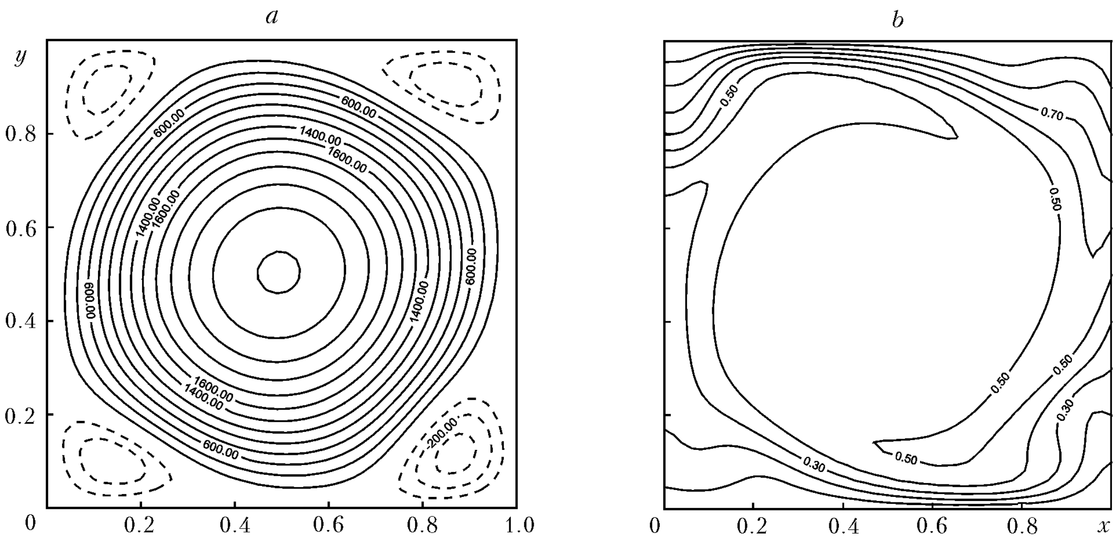


Fig. 1. Stream-function (a) and temperature (b) fields in rotation of the vector of free fall acceleration with a resonant frequency. $Gr_T = 1.1 \cdot 10^8$, Pr = 0.018, and $F_{rot} = 2500$.

results of the existing solutions of the test problem and with the data of the corresponding experiment has shown high accuracy and efficiency of the computer program. The scheme was used earlier for solution of different convection problems [5, 8]. In these publications, it was shown that one can obtain solutions for velocities corresponding to Reynolds numbers of ~30,000 on fairly coarse grids and with large time steps. Further improvement of the computational scheme for solution of the problem formulated involved changing the boundary conditions in accordance with the description presented above. To determine the oscillation spectra of the parameters in the liquid we used the Hanning algorithm of Fourier analysis.

2. Calculation Results. *Thermal Gravitational Convection.* Thermal gravitational convection was studied for a square cavity and liquids with Prandtl numbers equal to 1 and 0.018. The structure of flow in a rotating field of free fall acceleration depends on the value and rotational frequency of the vector g. At high (but lower than the "acoustic" frequency $f_* = v_* L^{-1}$) rotational frequencies of the vector of free fall acceleration, the particles of the liquid vibrate with a frequency, executing a drift with a low velocity corresponding to the intensity of the average vibrational convection. At low frequencies, flow is disordered and consists of elements inherent in steady-state flow for certain static positions of the acceleration vector. The regime of resonant flow is between these limits. The flow structure and the temperature field for this regime are shown in Fig. 1. The central part of the liquid is isothermal, in practice, and rotates as a solid body. All the parameters in the liquid oscillate in this regime with a resonant frequency, which is seen in Fig. 2. The instant at which the resonant regime of flow begins, when the rotational frequency of the vector coincides with the eigenfrequency of the convective cell, can accurately be determined from the maximum of a plot showing the time-average value of the maximum of the stream function against the rotational frequency of the vector of gravitational acceleration g. Such dependences for three values of the Rayleigh number are presented in Fig. 3. Time averaging has been performed, since this parameter also oscillates with a small amplitude. It is clear from the figure that the value of the resonance effect under study decreases with acceleration amplitude. That is the reason why it is very small under vibration aboard the spacecraft and cannot be measured.

Figure 4 shows the resonant rotational frequencies of the acceleration vector g as functions of the Rayleigh number for liquids with Prandtl numbers equal to 1 ad 0.018.

Direct numerical calculations of oscillating thermal gravitational convection in the square cavity were carried out with the same boundary conditions as those used in obtaining generalization for the eigenfrequencies of the convective cell. For Pr = 0.018 and $Gr_T = 2.2 \cdot 10^7$ and $4.4 \cdot 10^7$ the oscillation frequencies of the parameters in the liquid turned out to be equal to 1100 and 1700 respectively. Temperature variations at a point with the coordinates $x = 0.27$, $y = 0.72$ are shown as an example in Fig. 5 for $Gr_T = 2.2 \cdot 10^7$. The generalized dependence (Fig. 4) gives eigenfre-

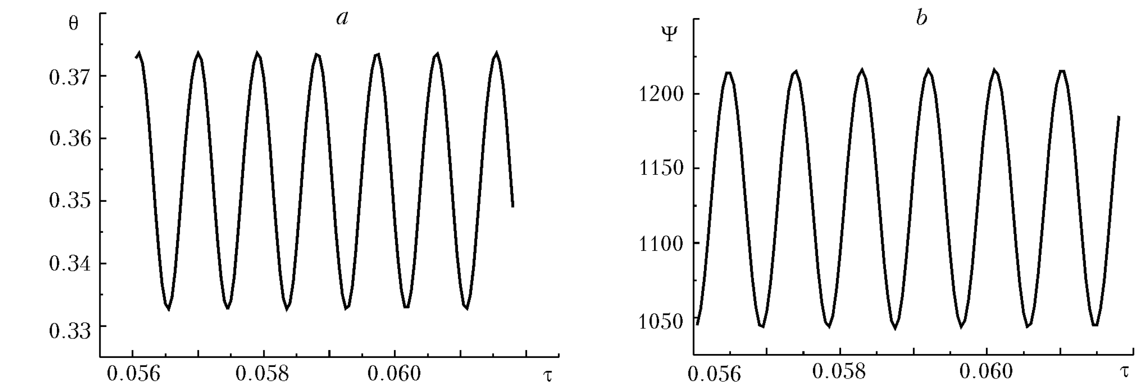
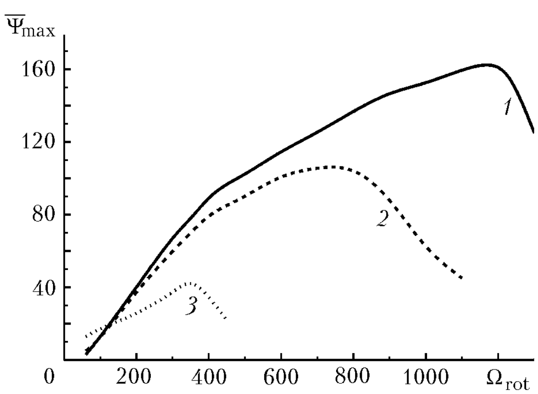


Fig. 2. Variation in the temperature at the selected point x = 0.27, y = 0.72 (a) and in the maximum value of the stream function (b) at the instant of resonance. Gr_T = 2.2·10^7, Pr = 0.018, and F_{rot} = 1100.



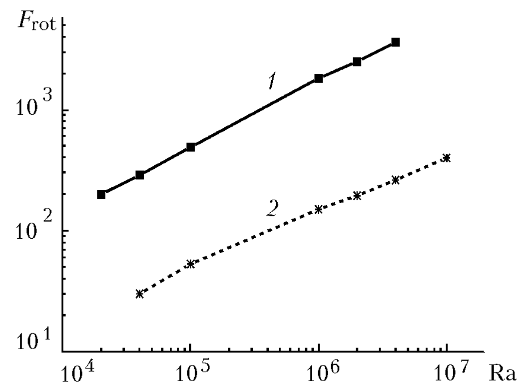


Fig. 3. Time-average maximum of the stream function vs. rotational frequency of the free-fall-acceleration vector for three values of the Rayleigh number: 1) Ra = 1.7·10^6, 2) 7·10^5, and 3) 1.7·10^5. Pr = 1.

Fig. 4. Generalized dependence of the resonant rotational frequency of the vector of free fall acceleration on the Rayleigh number for thermal gravitational convection in a square cavity: 1) Pr = 0.018 and 2) 1.0.

quencies of the convective cell of 1090 and 1650 for these Grashof numbers. Thus, the assumption on the identity of the resonant frequencies for convective cells and the frequencies of oscillatory convection was confirmed.

We also compared the resonant frequency and the temperature-oscillation frequency obtained in the experiment of Boyarevich and Gorbunov [12]. In this experiment an In-Ga-Sn melt (Pr = 0.018) was placed in a cell in the shape of a short parallelepiped with a square cross section. Thermal boundary conditions were the same as those used in our calculations. The temperature at a point with the coordinates x = 0.27, y = 0.82 in the square cross section drawn through the center of the short sides of the parallelepiped oscillated at a dominant dimensionless frequency of ~1200 for Gr_T = 2.2·10^7, when there was no magnetic field in the experiment. Thus, the difference of the experimental frequency from the calculated values (1100 and 1200) does not exceed 10%.

The dependence of the eigenfrequency (oscillation frequency) on the Rayleigh number in the Ra range between 1·10^4 and 10^6 for a Prandtl number equal to 0.018 can be expressed in the form F_{res} = 0.67 $\text{Ra}^{0.574}$ in accordance with the generalization presented in Fig. 4. The exponent in this relation very closely corresponds to the exponent for the similar relation (0.662) that has been obtained from the data presented in Müller's monograph [3]. The difference in the preexponential factors (0.67 from our data and 0.2 from the data of [3, p. 104]) is attributable to another geometry (cylinder) and somewhat different thermal boundary conditions. In what follows we will consider the dependence of the preexponential factor on the geometry of a liquid volume in greater detail.

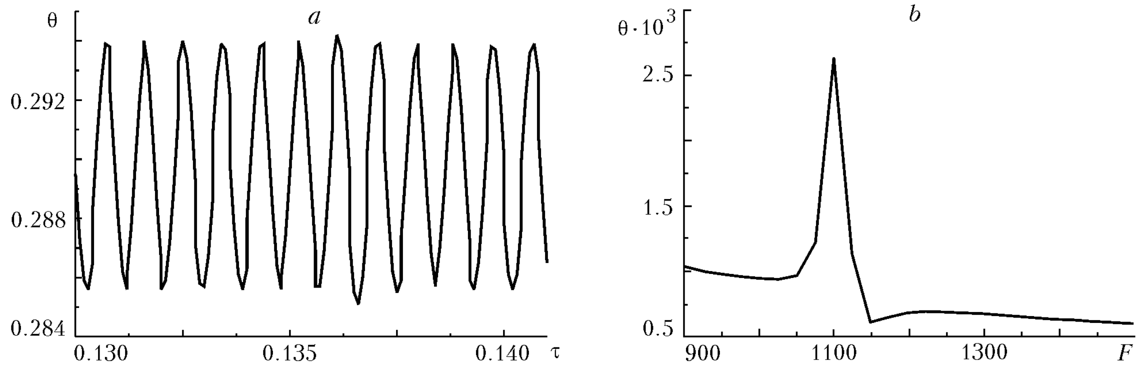
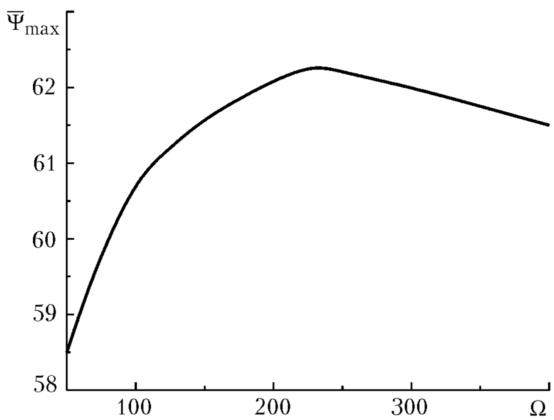


Fig. 5. Results of calculation of oscillating thermal gravitational convection in a square cavity: a) temperature oscillations at the point x = 0.27, y = 0.72; b) spectrum of these oscillations. $\mathrm{Gr}_T = 2.2 \cdot 10^7$, Pr = 0.018.



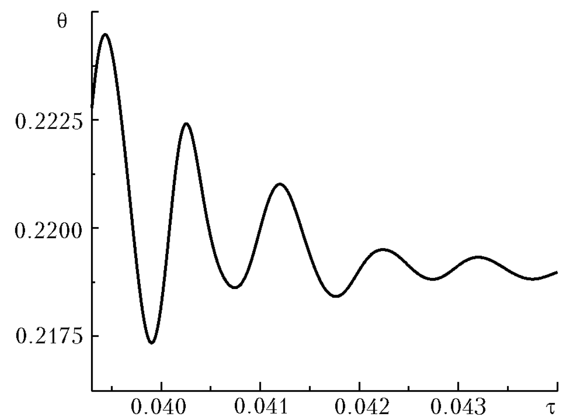


Fig. 6. Time-average maximum of the stream function vs. oscillation frequency of the heat flux at the free boundary of the liquid in a rectangle. Ma = $4.4 \cdot 10^2$, L/d = 2, A = 0.8, B = 0.3, and A_1 = 0.6.

Fig. 7. Damped oscillations of temperature at a point on the free surface of a liquid column after the action of a shock heat flux at this boundary at the point x = 1.0, y = 0.5. Ma = $9 \cdot 10^3$, A = 0.8, B = 0.3, and $\Omega_{imp} = 1.5 \cdot 10^4$.

Thermocapillary Convection. The eigenfrequencies of convective cells for liquid flows under the action of the thermocapillary effect were determined for rectangular and cylindrical regions with different degrees of elongation: *L/d* and *L/R* respectively. Furthermore, we considered two forms of distribution of the heat flux on the free surface of the liquid for L/d = 2, which differed in the values of the numerical parameters A and B. In the calculations, the Prandtl number was equal to 0.018.

The process of determination of the resonant frequency under the action of the time-variable heat flux on the free surface is very close to that considered above for thermal gravitational convection. In this case, too, a small variable heat flux is added to a constant heat flux creating thermocapillary flow. We see the oscillation frequency at which there is a maximum in the dependence $\overline{\psi}_{max}$ (f). It is precisely this frequency that is the resonant frequency (or eigenfrequency) of the convective cell, which is clearly shown by Fig. 6.

Also, we used the procedure of shock action of the heat flux in accordance with relation (5). Figure 7 shows the reaction of the liquid to the action of such a thermal load. After random oscillations (most of this stage is not presented in this figure), the damped oscillations of the parameters in the liquid are executed at eigenfrequencies for convective cells. We note that when both procedures are used, the oscillation spectrum of the parameters in the liquid contains the frequency of action applied which is excluded in spectral analysis.

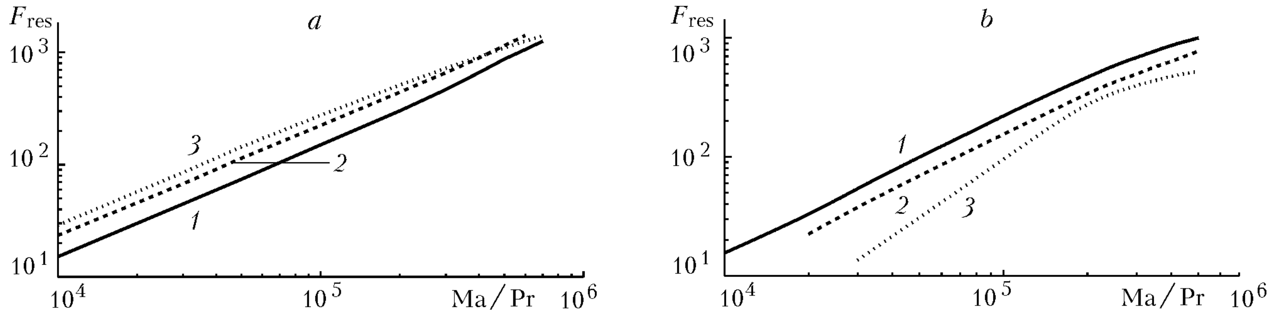


Fig. 8. Generalized dependence of the resonant frequency on the complex parameter for thermocapillary convection in a rectangular cavity (a) (1) L/d = 2, A = 0.8, and B = 0.3; 2) 3, 0.8, and 0.3; 3) 2, 2.7, and 8) and in a liquid cylinder with three values of the elongation (b) (1) L/R = 2, 2) 2.5, and 3) 3 (A = 0.8 and B = 0.3).

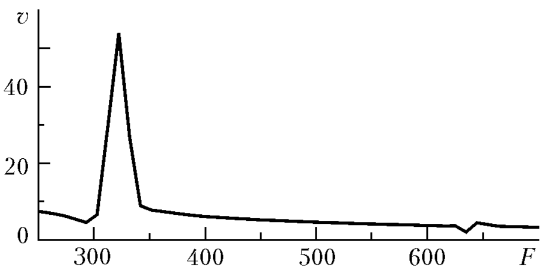


Fig. 9. Oscillation spectrum of the dimensionless velocity at the point r = 1, z = 1.2 on the free surface of a liquid column. Ma = $4{\cdot}10^3$, Pr = 0.018, L/R = 3, A = 0.8, and B = 0.3.

Figure 8 gives the dependences of the resonant frequency (eigenfrequency of the convective cell) on the complex parameter Ma/Pr frequently used to characterize the intensity of thermocapillary convection. We note that in the region of low flow velocities, when Ma/Pr $< 2{\cdot}10^5$, and for a ratio of the sides lower than 3 the slope of the curves is the same for the rectangular and cylindrical geometries of the liquid, although the absolute values for the same elongations differ for the rectangle and the cylinder. This fact can serve as an indirect explanation for the result of the comparison of the data obtained in the present investigation and those given in [3].

We carried out direct calculations of oscillating thermocapillary convection and compared the results to the generalizations obtained for the eigenfrequencies of the convective cell. Figure 9 shows the spectrum of velocity oscillations at a point on the free cylindrical surface of the liquid column of elongation L/R = 3 and for heat-flux distribution containing constants A and B of 0.8 and 0.3 respectively at this boundary. In this case the Marangoni and Prandtl numbers were $4{\cdot}10^3$ and 0.018 respectively. The spectrum contains a frequency of 322, whereas the generalizing curve shown in Fig. 8b gives a value of the resonant frequency F_{res} of 310.

No comparison to experimental data was carried out, since they are incomplete. First of all, we failed to measure the heat flux on the free surface in the experiments or at least the temperature distributions at this boundary, which would allow approximate judgment of this boundary condition. Furthermore, the experiments, as has been indicated in the Introduction, were carried out for liquids having Prandtl number Pr >> 1 and without measuring the vibrational situation near the experimental setup, which could distort the frequency characteristics of parametric oscillations in the liquid quite significantly.

CONCLUSIONS

The investigation carried out has shown that the introduced notion "eigenfrequency of a convective cell" can be applied to determination of the oscillation frequencies of thermal gravitational and thermocapillary convection. Knowing the oscillation spectrum of convective cells, in the analysis of experiments in which the oscillations of the parameters in the liquid are observed, we can single out the oscillations that are not related to the hydrodynamic instability of flow and thus are only the reaction of the medium to the external action and not the characteristics proper of oscillating (turbulent) convection. The above reason is particularly significant for conditions under which the experiments aboard spacecraft were carried out. We should take into account the phenomenon found in [4, 5] where the oscillation of the free surface in the presence of deformation is executed with a frequency lower than the vibration frequency, and the oscillation spectra of the parameters in the liquid contain additional frequencies that are due to the nonlinearity of the medium and different characteristic velocities for the transfer of heat and mass, when the Prandtl and Schmidt numbers are different from unity.

In this investigation, we have presented results of determination of the eigenfrequencies for convective structures consisting of one cell or for symmetric structures consisting of two identical cells (in the case of thermocapillary convection with a symmetric distribution of the heat flux on the free surface). For more complex structures, several eigenfrequencies and combined frequencies must be present in the spectrum under study. Investigation of these more complex cases can be carried out with the use of the procedures proposed.

The use of the eigenfrequency of a convective cell provides an explanation for the change in the oscillation frequencies of convection upon the application of a magnetic field, since it clearly demonstrates the manner in which this effect is related to the change in the flow structure and in the velocities of flow of the liquid in the magnetic field. The quantitative value of the eigenfrequency of the convective cell reveals a clear boundary dividing the regimes of vibrational convection (hydrodynamically high frequencies) and of irregular convection (low frequencies) in the field of time-variable accelerations of body forces, for example, gravity.

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NOTATION

a, thermal diffusivity of the liquid, $m^2 \cdot sec^{-1}$; A, B, and A_1, B_1, constants in expressions (4) and (5); d, width of the rectangular liquid region, m; **e**, dimensionless vector with the coordinates of the buoyancy force in (2); f, frequency, sec^{-1}; $F = fR^2\nu^{-1}$ (or $fd^2\nu^{-1}$), dimensionless frequency; g, free fall acceleration, $m \cdot sec^{-2}$; $Gr_T = g\beta_T \Delta T d^3 \nu^{-2}$, Grashof number for thermal convection; L, length of the liquid region, m; L/R (or L/d), elongation of the liquid region per unit length; $Ma = -(\partial\sigma/\partial T)R(\text{or } d)\Delta T(\rho\nu a)^{-1}$, Marangoni number; $Pr = \nu a^{-1}$, Prandtl number; p, pressure, Pa; r, z, cylindrical coordinates, m; R, radius of the cylindrical liquid region, m; $Ra = Gr_T Pr$, Rayleigh number; T, temperature, K; t, time, sec; **v**, velocity vector, $m \cdot sec^{-1}$; x, y, rectangular coordinates, m; β_T, coefficient of thermal expansion of the liquid, K^{-1}; $\Delta T = T_{max} - T_0$, temperature difference, K; $\theta = (T - T_0)/\Delta T$, dimensionless temperature; ν, kinematic viscosity, $m^2 \cdot sec$; ρ, density of the liquid, $kg \cdot m^{-3}$; σ, coefficient of surface tension of the liquid, $N \cdot m^{-1}$; $\tau = tR^2(\text{or } d^2)\nu$, dimensionless time; ψ, dimensionless stream function; $\omega = 2\pi f$, circular frequency, sec^{-1}; $\Omega = \omega R^2(\text{or } d^2)\nu^{-1}$, dimensionless circular frequency. Subscripts: 0, initial temperature in the system; g points to the gravitation effect; imp, impulse; max, maximum value of the parameter; res, parameter referring to resonance; rot, rotational; x, y, and r, projections of the parameter onto the coordinate axes; *, quantities referring to the acoustic region of the liquid flow; bar, averaging of the parameter over the frequency of action applied.

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